1

$$x_1 + 2x_2 + 5x_3 = 0$$
$$2x_1 + x_3 = 2$$
$$-x_1 + x_2 + 3x_3 = 1$$

denklem sistemini Gauss Eleminasyon yöntemiyle çözün

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 0 \\ 2 & 0 & 1 & 2 \\ -1 & 1 & 3 & 1 \end{array}\right]$$

$R_2 \to R_2 + 2R_3$
$R_3 \to R_1 + R_3$

$$\begin{bmatrix} 1 & 2 & 5 & 0 \\ 0 & 2 & 7 & 4 \\ 0 & 3 & 8 & 1 \end{bmatrix}$$

$R_2 \to R_2/2$
$R_3 \to R_3 - R_2(3/2)$

$$\begin{bmatrix} 1 & 2 & 5 & 0 \\ 0 & 1 & 7/2 & 2 \\ 0 & 0 & -5/2 & -5 \end{bmatrix}$$

$$-\frac{5}{2} x_3 = -5 \Rightarrow x_3 = 2$$

$$x_2 - \frac{7}{2} x_3 = 2$$

$$x_2 - \frac{7}{2} \cdot 2 = 2 \Rightarrow x_2 = -5$$

$$x_1 + 2x_2 + 5x_3 = 0$$

$$x_1 + 2\cdot(-5) + 5\,2 = 0 \Rightarrow x_1 = 0$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ 2 \end{bmatrix}$$

(2) $$\begin{bmatrix} 0 & 0 & 3 \\ 0 & 1 & 4 \\ 1 & 2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 0 \end{bmatrix}$$

Denklem sistemini Gauss-Jordan yöntemiyle çözün.

$$\left[\begin{array}{ccc|c} 0 & 0 & 3 & 6 \\ 0 & 1 & 4 & 3 \\ 1 & 2 & 5 & 0 \end{array}\right]$$

$R_1 \to R_3$
$R_3 \to R_1/3$
$$\begin{bmatrix} 1 & 2 & 5 & 0 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$R_2 \to R_2 - 4R_1$
$$\begin{bmatrix} 1 & 2 & 5 & 0 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$R_1 \to R_1 - 2R_2 - 5R_3$
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ 2 \end{bmatrix}$$

③ $2x_2 + 3x_3 = 5$
$2x_1 + 6x_3 = 4$
$3x_1 + 2x_2 = 7$ denklem sistemini Kramer yöntemiyle çözün.

$$A = \begin{bmatrix} 0 & 2 & 3 \\ 2 & 0 & 6 \\ 3 & 2 & 0 \end{bmatrix}$$

$$\det(A) = 2.6.3 + 3.2.2 = 48$$

$$\det(A_1) = \begin{vmatrix} 5 & 2 & 3 \\ 4 & 0 & 6 \\ 7 & 2 & 0 \end{vmatrix} = 7.2.6 + 3.4.2 - 5.2.6 = 48$$

$$x_1 = \frac{\det(A_1)}{\det A} = \frac{48}{48} = 1$$

$$\det(A_2) = \begin{vmatrix} 0 & 5 & 3 \\ 2 & 4 & 6 \\ 3 & 7 & 0 \end{vmatrix} = 3.5.6 + 3.2.7 - 3.4.3 = 96$$

$$x_2 = \frac{\det(A_2)}{\det(A)} = \frac{96}{48} = 2$$

$$\det(A_3) = \begin{vmatrix} 0 & 2 & 5 \\ 2 & 0 & 4 \\ 3 & 2 & 7 \end{vmatrix} = 3.2.4 + 5.2.2 - 7.2.2 = 16$$

$$x_3 = \frac{\det(A_3)}{\det(A)} = \frac{16}{48} = \frac{1}{3}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1/3 \end{bmatrix}$$

(4) $A = \begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 7 & 8 & 9 \end{bmatrix}$ matrisinin determinantını kofaktör açılımıyla elde ediniz.

$$\det(A) = a_{11} C_{11} + a_{12} C_{12} + a_{13} C_{13}$$

$$C_{11} = (-1)^{1+1} M_{11} = 1 \cdot \begin{vmatrix} 5 & 4 \\ 8 & 9 \end{vmatrix} = 5.9 - 8.4 = 13$$

$$C_{12} = (-1)^{1+2} \cdot M_{12} = -1 \begin{vmatrix} 6 & 4 \\ 7 & 9 \end{vmatrix} = -(6.9 - 7.4) = -26$$

$$C_{13} = (-1)^{1+3} \; M_{13} = 1 \cdot \begin{vmatrix} 6 & 5 \\ 7 & 8 \end{vmatrix} = 6.8 - 7.5 = 13$$

$$\det(A) = 1.13 + 2.(-26) + 3.13 = 0$$

(5)

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 2 & 1 \\ 1 & 1 & 3 \end{bmatrix}$$ matrisinin eigen değerlerini elde ediniz.

$$\lambda I - A = \begin{bmatrix} \lambda-1 & -2 & 1 \\ -3 & \lambda-2 & -1 \\ -1 & -1 & \lambda-3 \end{bmatrix}$$

$$\det(\lambda I - A) = \begin{vmatrix} \lambda-1 & -2 & 1 \\ -3 & \lambda-2 & -1 \\ -1 & -1 & \lambda-3 \end{vmatrix} = 0$$

$$= (\lambda-1)\begin{vmatrix} \lambda-2 & -1 \\ -1 & \lambda-3 \end{vmatrix} - (-2)\begin{vmatrix} -3 & -1 \\ -1 & \lambda-3 \end{vmatrix} + 1.\begin{vmatrix} -3 & \lambda-2 \\ -1 & -1 \end{vmatrix}$$

$$= (\lambda-1)\left[(\lambda-2)(\lambda-3)-1\right] + 2\left[(-3)(\lambda-3)-1\right] + \left[(-3)(-1)-(-1)(\lambda-2)\right]$$

$$= \lambda^3 - 5\lambda^2 + 5\lambda - \lambda^2 + 5\lambda - 5 - 6\lambda + 16 + 3 + \lambda - 2$$

$$= \lambda^3 - 6\lambda^2 + 5\lambda + 12$$

$$= (\lambda+1)(\lambda^2 - 7\lambda + 12)$$

$$= (\lambda+1)(\lambda-3)(\lambda-4) = 0$$

$$\begin{array}{l} \lambda^3 - 6\lambda^2 + 5\lambda + 12 \\ \lambda^3 + \lambda^2 \\ \quad -7\lambda^2 - 7\lambda \\ \qquad +12\lambda + 12 \end{array}$$

$\lambda_1 = -1$

$\lambda_2 = 3$

$\lambda_3 = 4$

⑥ $\vec{a} = \vec{i} + 2\vec{j} - 3\vec{k}$

$\vec{b} = 3\vec{i} - 2\vec{j} + \vec{k}$

vektörleri için

$(2\vec{a}+\vec{b}) \times (\vec{a} - 2\vec{b})$ işleminin sonucunu bulunuz.

$2\vec{a}+\vec{b} = 2.(\vec{i}+2\vec{j}-3\vec{k}) + (3\vec{i}-2\vec{j}+\vec{k})$

$= 5\vec{i} + 2\vec{j} - 5\vec{k}$

$\vec{a} - 2\vec{b} = (\vec{i} + 2\vec{j} - 3\vec{k}) - 2(3\vec{i} - 2\vec{j} + \vec{k})$

$= -5\vec{i} + 6\vec{j} - 5\vec{k}$

$(2\vec{a}+\vec{b}) \times (\vec{a}-2\vec{b}) = (5\vec{i} + 2\vec{j} - 5\vec{k})(-5\vec{i} + 6\vec{j} - 5\vec{k})$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & 2 & -5 \\ -5 & 6 & -5 \end{vmatrix}$$

$$= \vec{i}\begin{vmatrix} 2 & -5 \\ 6 & -5 \end{vmatrix} - \vec{j}\begin{vmatrix} 5 & -5 \\ -5 & -5 \end{vmatrix} + \vec{k}\begin{vmatrix} 5 & 2 \\ -5 & 6 \end{vmatrix}$$

$$= \vec{i}\left[2.(-5) - 6.(-5)\right] - \vec{j}\left[5.(-5) - (-5)(-5)\right] + \vec{k}\left[5.6 - (-5).2\right]$$

$$= 20\vec{i} + 50\vec{j} + 40\vec{k}$$